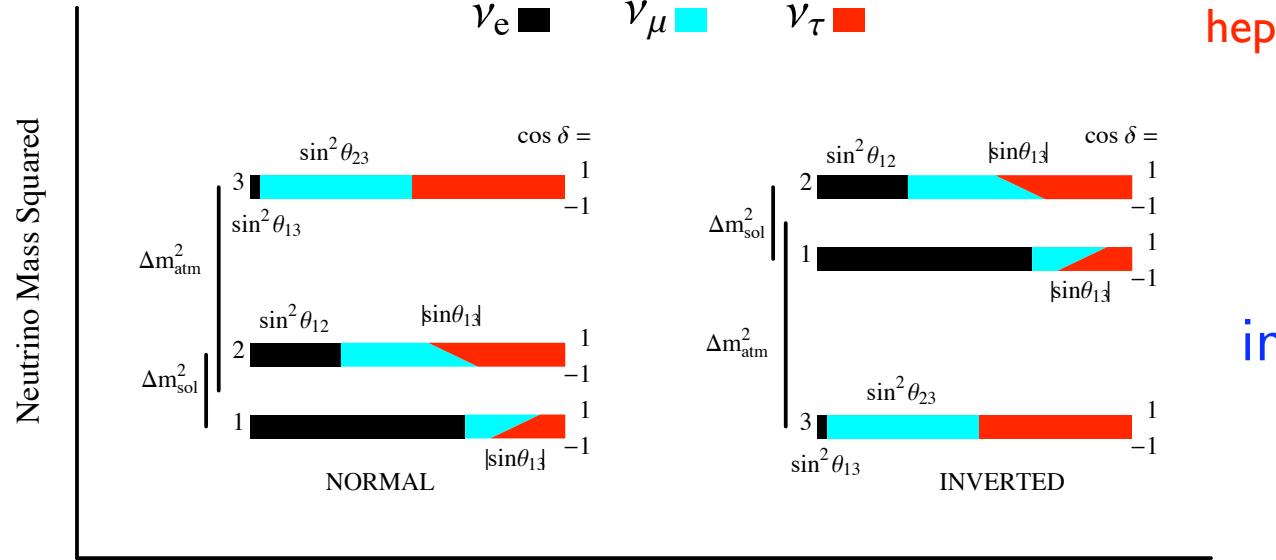


Ways to Determine the Neutrino Mass Hierarchy.

Stephen Parke - Fermilab
DOANOW: March 24, 2006

Mixings:

$$|U_{\alpha i}|^2$$



defn: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$

Fractional Flavor Content varying $\cos \delta$

CPT: $\delta \Leftrightarrow -\delta$ Invariant!

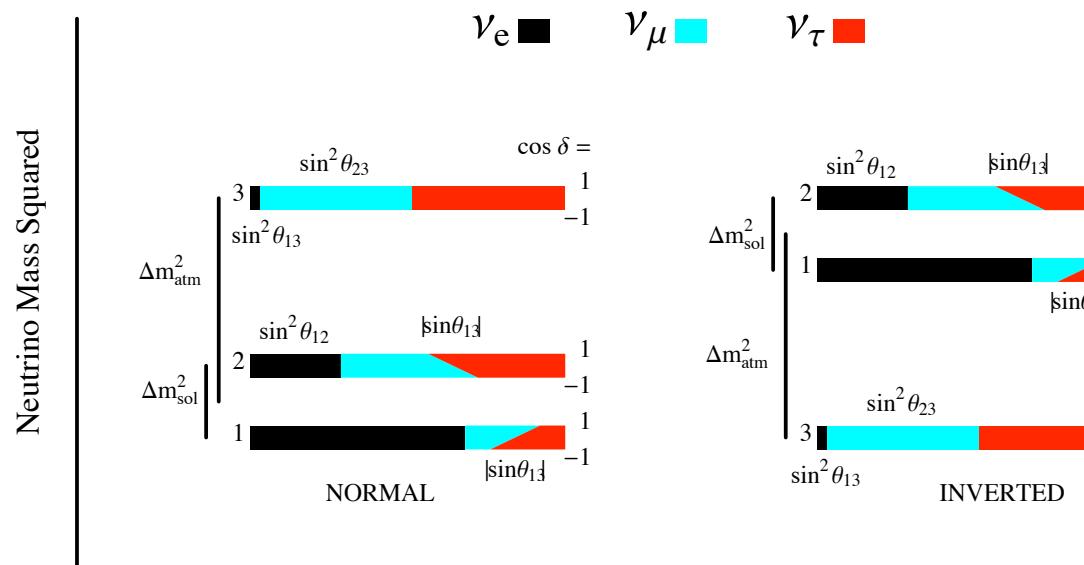
NH: $|\Delta m_{31}^2| > |\Delta m_{32}^2|$

IH: $|\Delta m_{31}^2| < |\Delta m_{32}^2|$

Less than
4% ν_e
in the 3 state!

Mixings:

$$|U_{\alpha i}|^2$$



defn: $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$

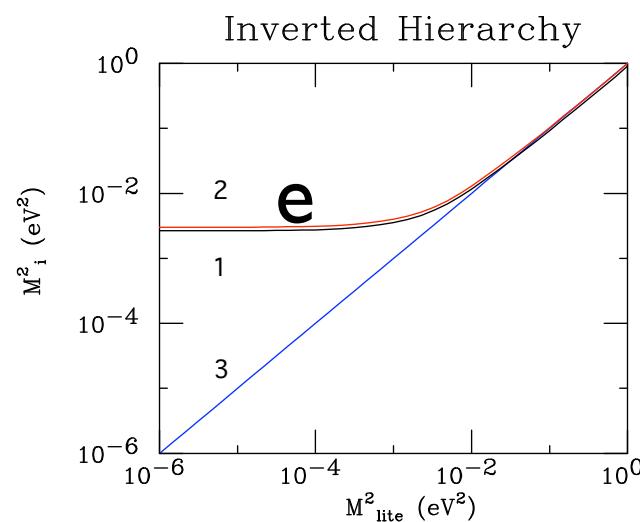
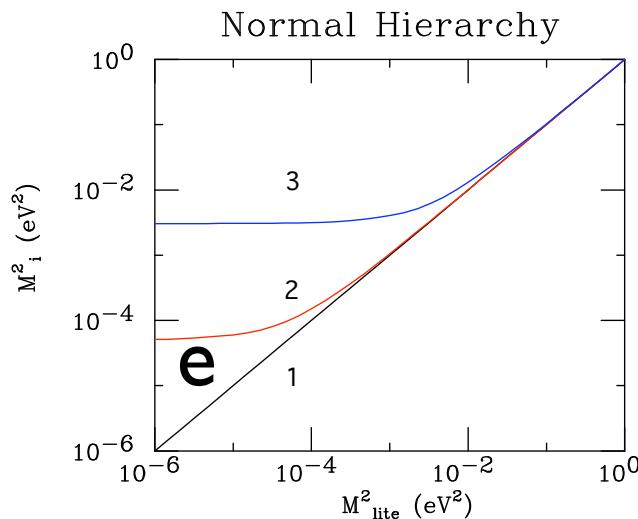
Fractional Flavor Content varying $\cos \delta$

CPT: $\delta \Leftrightarrow -\delta$ Invariant!

NH: $|\Delta m_{31}^2| > |\Delta m_{32}^2|$

IH: $|\Delta m_{31}^2| < |\Delta m_{32}^2|$

Masses:



States 1 and 2 are ν_e rich.

Less than
4% ν_e
in the 3 state!

Long Baseline $\nu_\mu \rightarrow \nu_e$ or $\nu_e \rightarrow \nu_\mu$

- SUPERBEAMS: (0.4 to 4 MW)
 - Counting Expts (3 ways)
 - Spectrum Measurement
- NEW NEUTRINO BEAMS
 - Neutrino Factory (muon storage ring)
 - High Gamma Beta Beams

Counting Expts near First Osc. Max.

T2K

JHF → Super-Kamiokande

- ✓ 295 km baseline
- ✓ Super-Kamiokande:
 - 22.5 kton fiducial
 - Excellent e/ μ ID
 - Additional π^0/e ID
- ✓ Hyper-Kamiokande
 - 20x fiducial mass of SuperK
- ✓ Matter effects small
- ✓ Study using fully simulated and reconstructed data



2.0, 2.5 or 3.0 deg

Counting Expts near First Osc. Max.

T2K

JHF → Super-Kamiokande

- 295 km baseline
- Super-Kamiokande:
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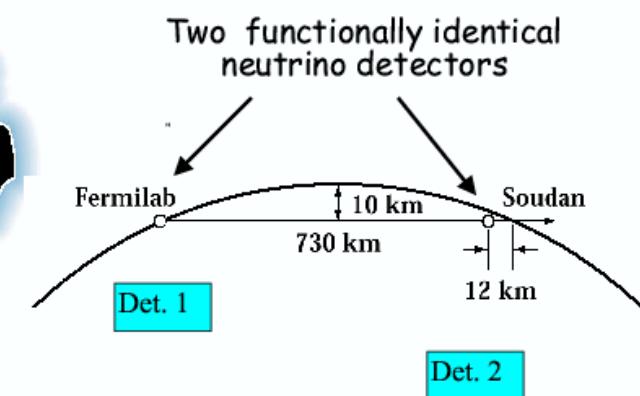


2.0, 2.5



The NUMI Beamline

NOVA

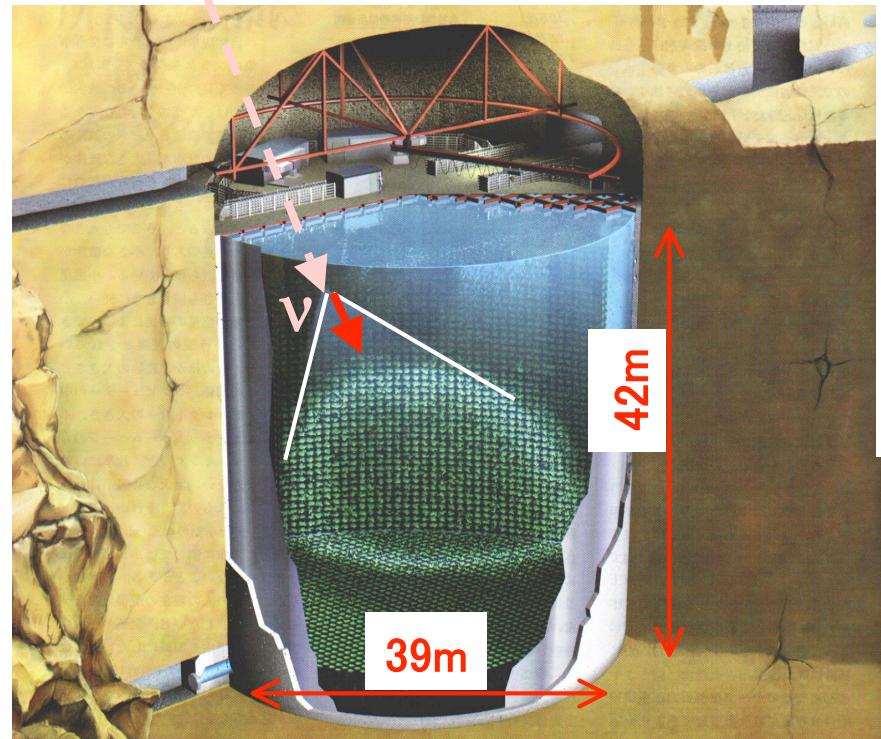


10, 12 14 km

Water Cherenkov detector



- 1996.4 Start data taking (SK- I)

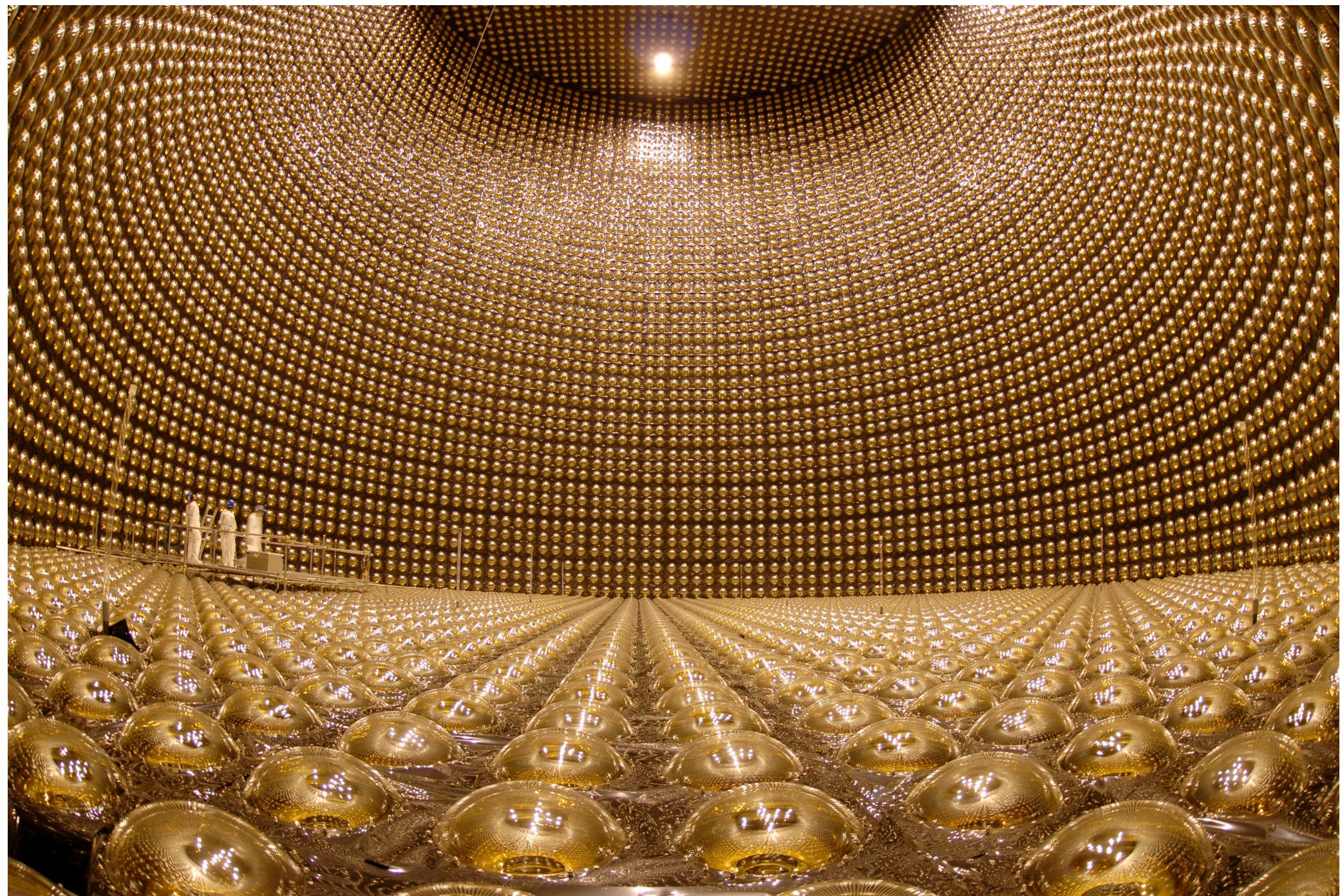


- 2700 w.e. overbuden
- 50,000 ton (22,500 ton fid.)
- 11,146 20 inch PMTs
- Photo cathode coverage: 40% of surface
- 1,885 anti-counter PMTs



- 2001.7 Stop data taking for detector upgrade
- 2001.11 Accident (6777 inner PMTs, 1100 outer PMTs were destroyed)
partial reconstruction of the detector
- 2002.10 Resume data taking (SK- II) mostly for K2K (photocathode coverage of 20%, 7MeV)
- 2005.10 Start full recovery work

Acrylic + FRP vessel



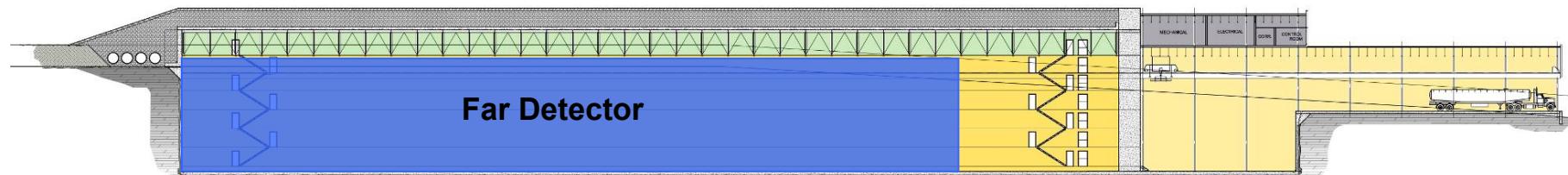
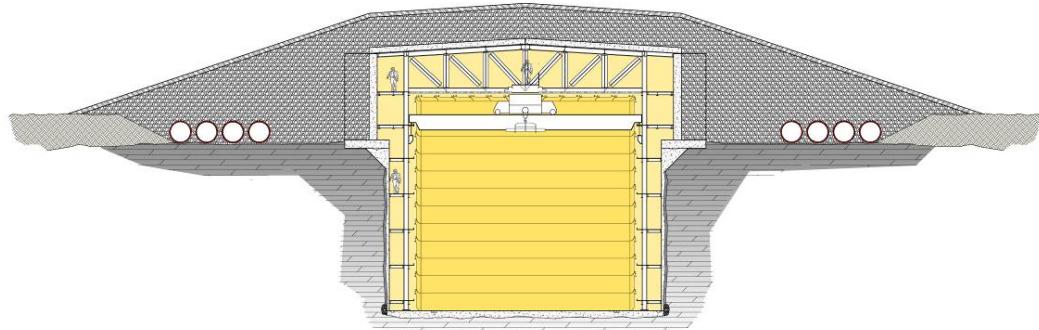
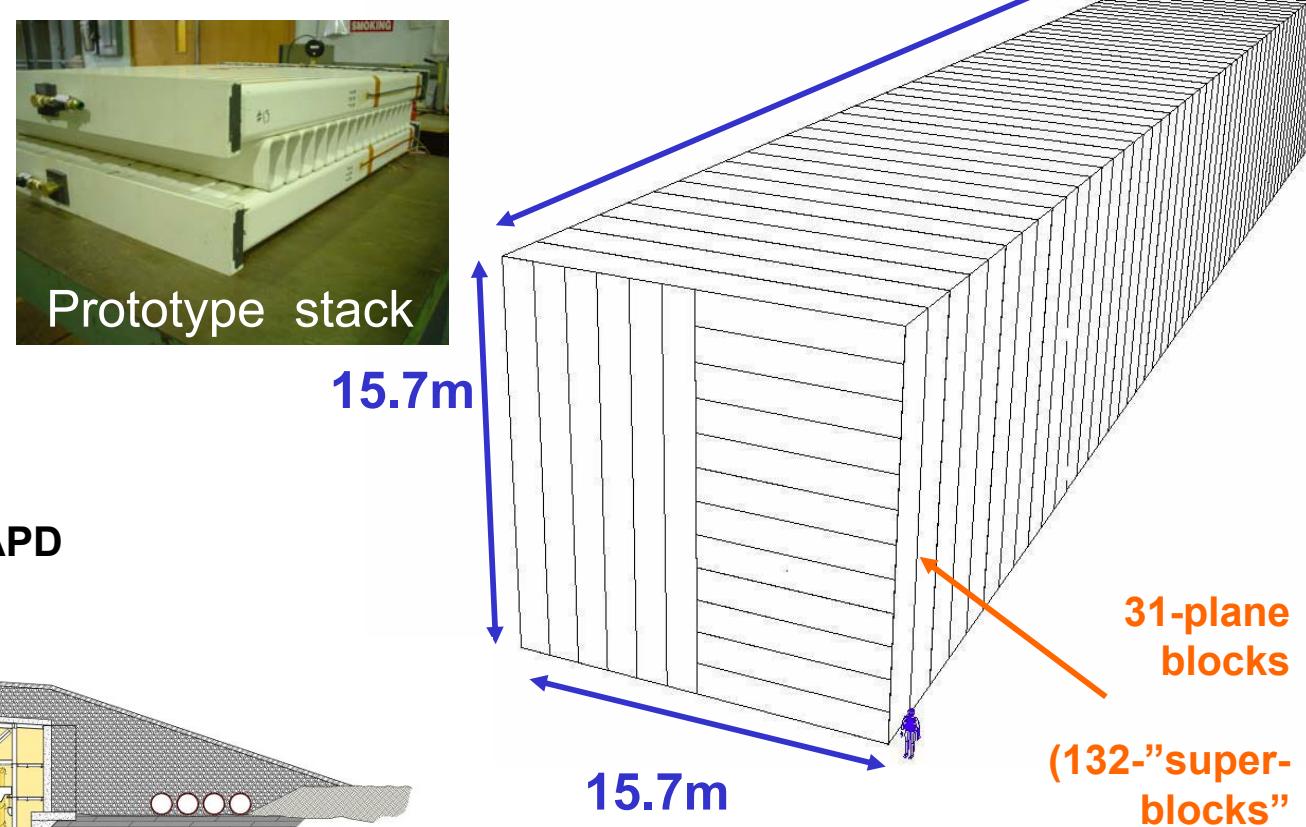


NOvA Far Detector we would like to build



- TAD = Totally Active Detector**
PVC = passive material
- mass N kT (N large)**
~80% scintillator
~20% PVC extrusions
- Modular structure**
32 cells/extrusion
12 extrusions/plane
1984 planes
- Cell dimensions:**
3.9 cm x 6 cm x 15.7m
- U-shaped 0.7 mm WLS fiber into APD**

We will build as much of this as the funding will allow...



$\nu_\mu \rightarrow \nu_e$ with MATTER

$\nu_\mu \rightarrow \nu_e$ with MATTER

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

$\nu_\mu \rightarrow \nu_e$ with MATTER

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

where $\sqrt{P_{atm}} = \sin \theta_{23} \sin 2\theta_{13} \frac{\sin(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)}$ Δ_{31}
in vac $\sin \Delta_{31}$

$\nu_\mu \rightarrow \nu_e$ with MATTER

$$P_{\mu \rightarrow e} \approx | \sqrt{P_{atm}} e^{-i(\Delta_{32} \pm \delta)} + \sqrt{P_{sol}} |^2$$

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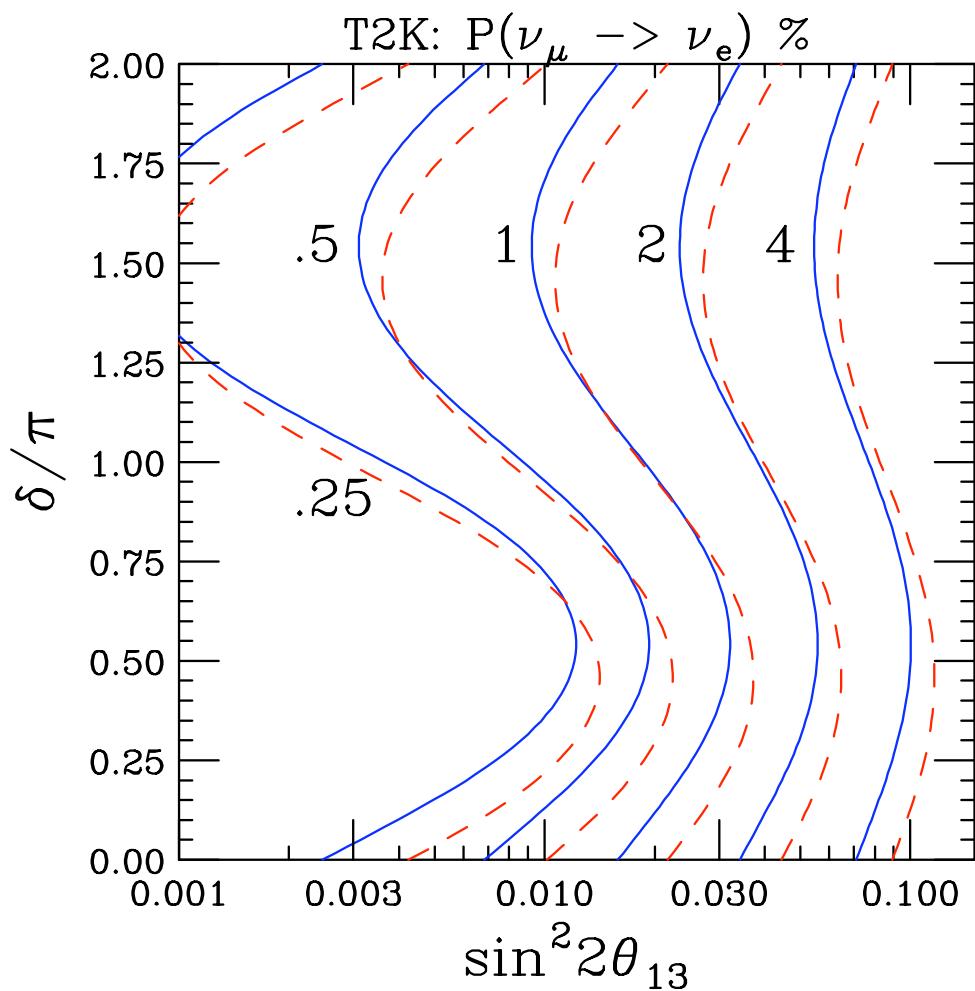
and $\sqrt{P_{sol}} = \cos \theta_{23} \sin 2\theta_{12} \frac{\sin(aL)}{(aL)}$ Δ_{21}
 in vac $\sin \Delta_{21}$

$$a = G_F N_e / \sqrt{2} = (4000 \text{ km})^{-1},$$

$$\pm = sign(\delta m_{31}^2) \quad \Delta_{ij} = |\delta m_{ij}^2| L / 4E$$

$\{\delta m^2 \sin 2\theta\}$ is invariant

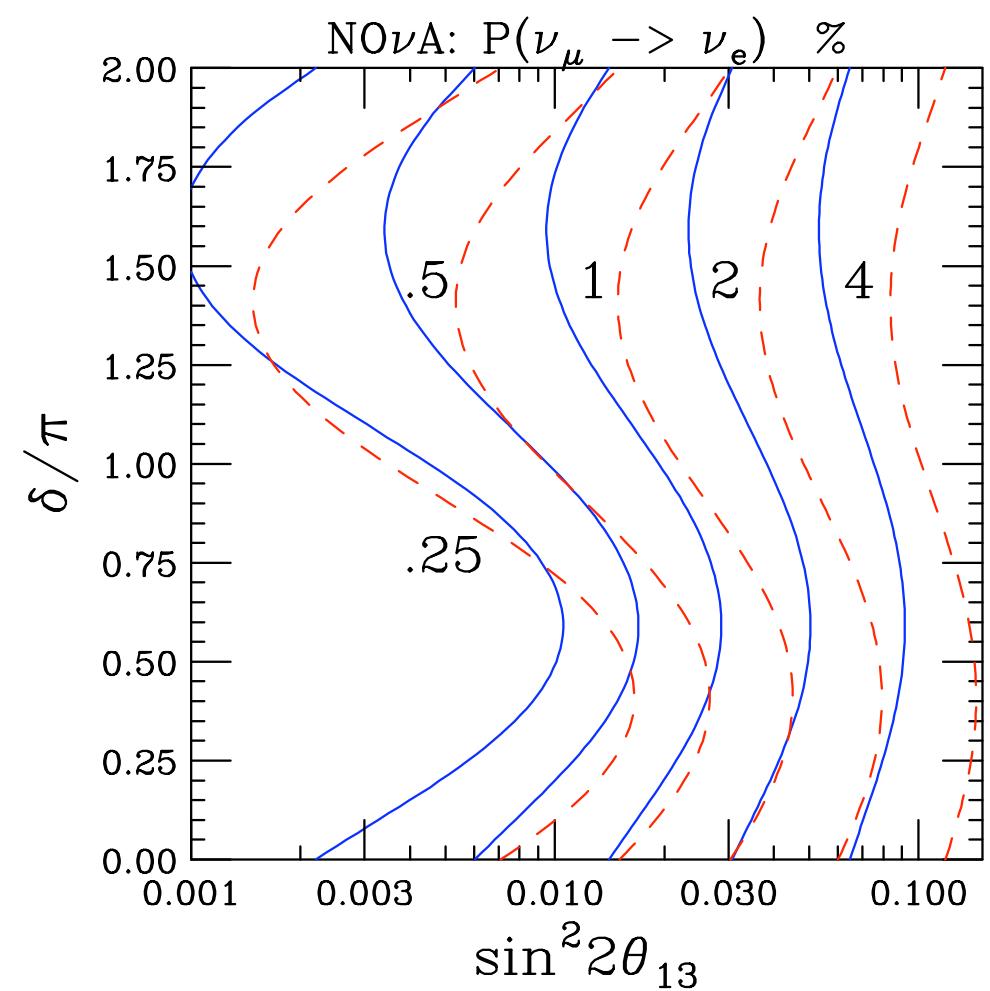
T2K:



$$\delta m_{31}^2 > 0$$

$$\delta m_{31}^2 < 0$$

NO ν A:

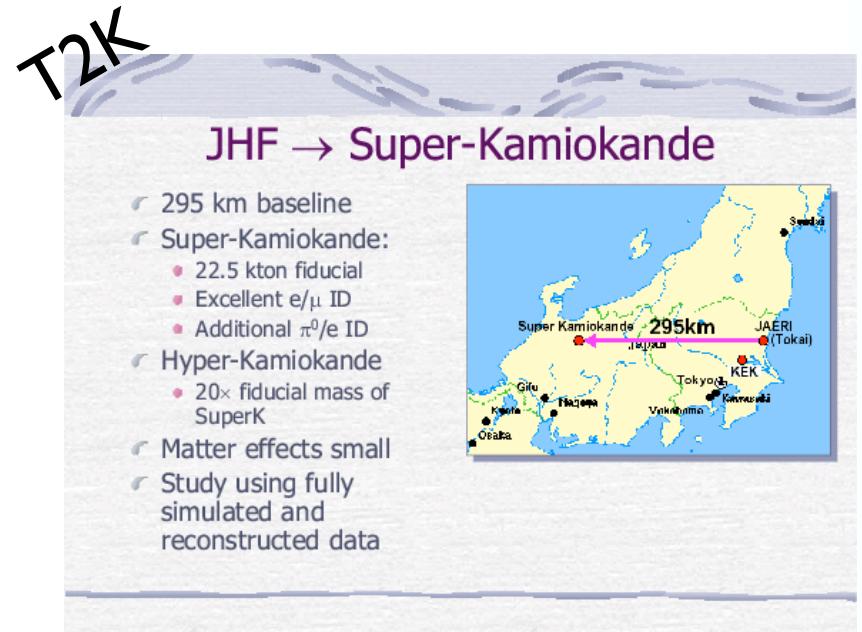
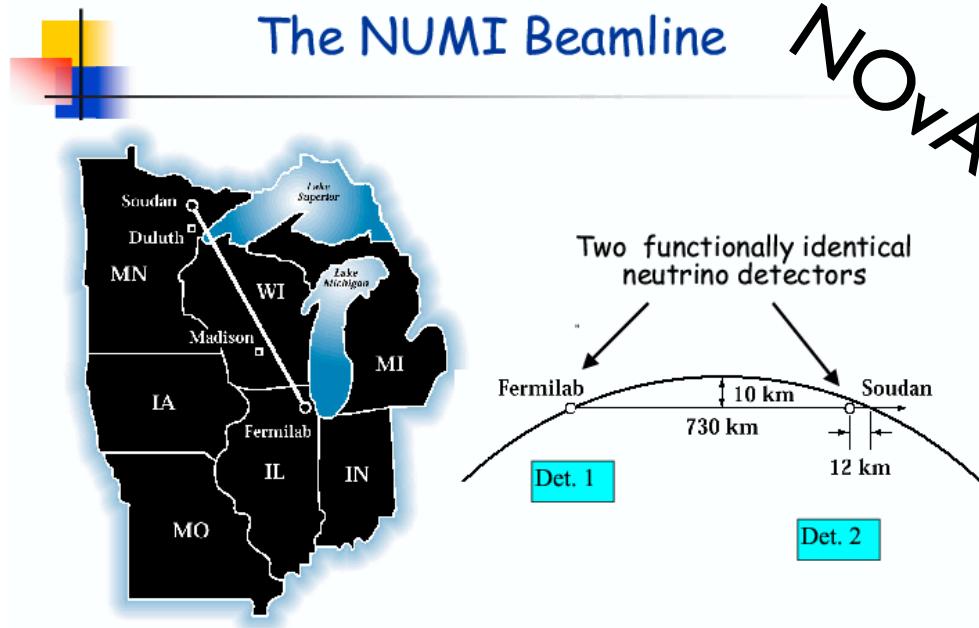


Phase I

Sensitivity approx 0.5%

Counting Expts at First Osc. Max.

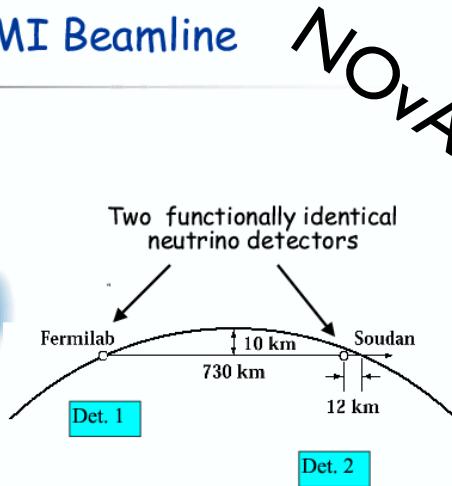
- Neutrino v Anti-Neutrino One Expt.
- Neutrino v Neutrino Two Expts Different L's and EQUAL E/L's
- Neutrino v Anti-Neutrino Two Expts Different L's



Neutrino v Anti-Neutrino One Expt.



The NUMI Beamline



NO_vA

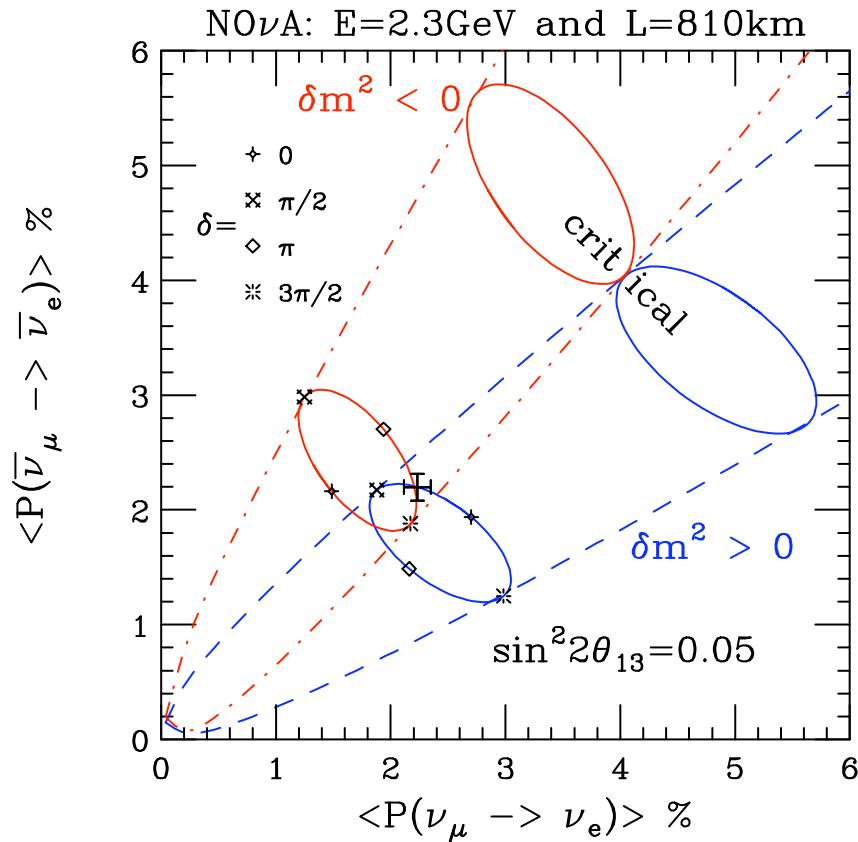
in the overlap region

$$\langle \sin \delta \rangle_+ - \langle \sin \delta \rangle_- = 2\langle \theta \rangle / \theta_{crit} \approx 1.4 \sqrt{\frac{\sin^2 2\theta_{13}}{0.05}}$$

exact along diagonal --- approximately true throughout the overlap region!!!

$$\theta_{crit} = \frac{\pi^2}{8} \frac{\sin 2\theta_{12}}{\tan \theta_{23}} \frac{\delta m_{21}^2}{\delta m_{31}^2} \left(\frac{4\Delta^2/\pi^2}{1-\Delta \cot \Delta} \right) / (aL) \sim 1/6$$

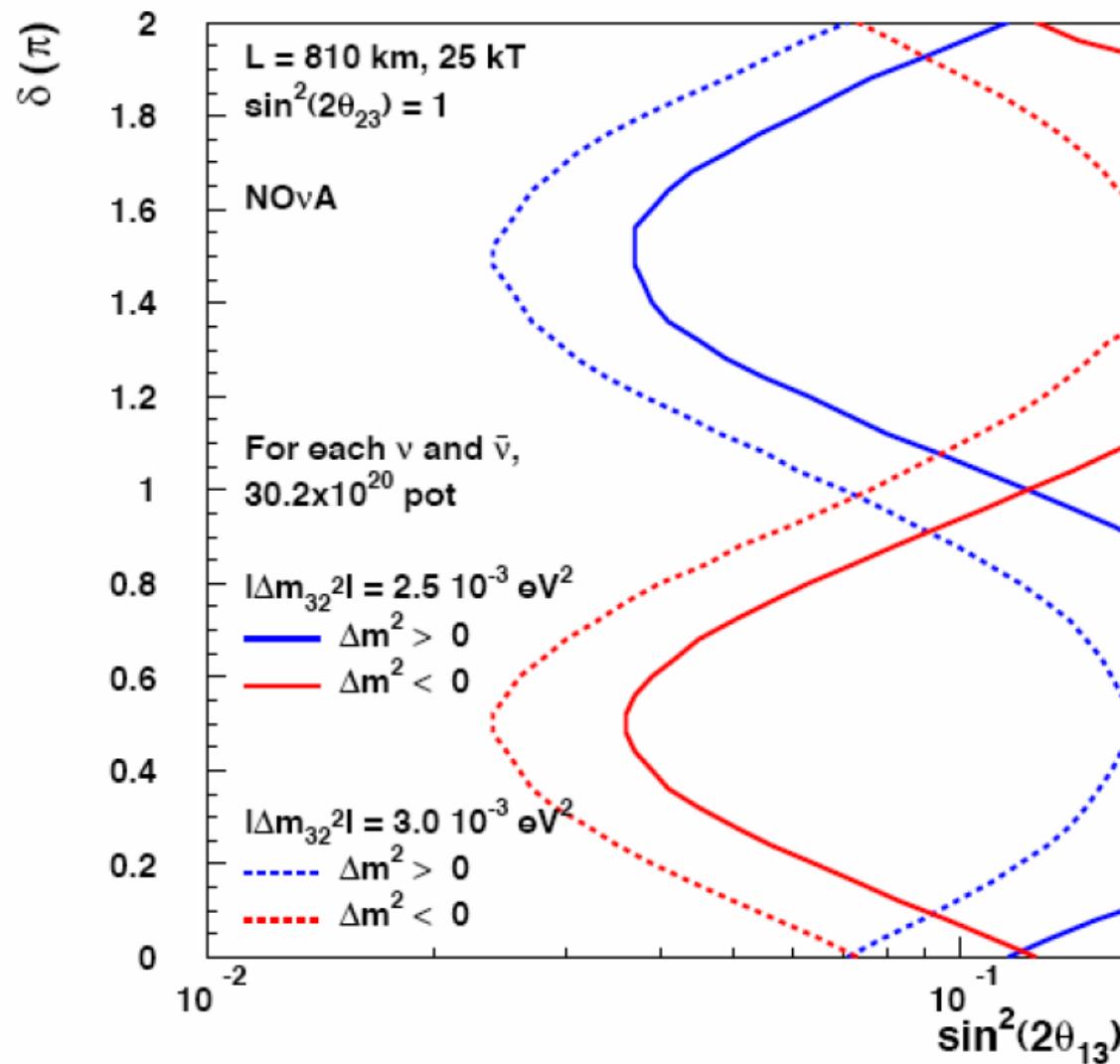
i.e. $\sin^2 2\theta_{crit} = 0.10$



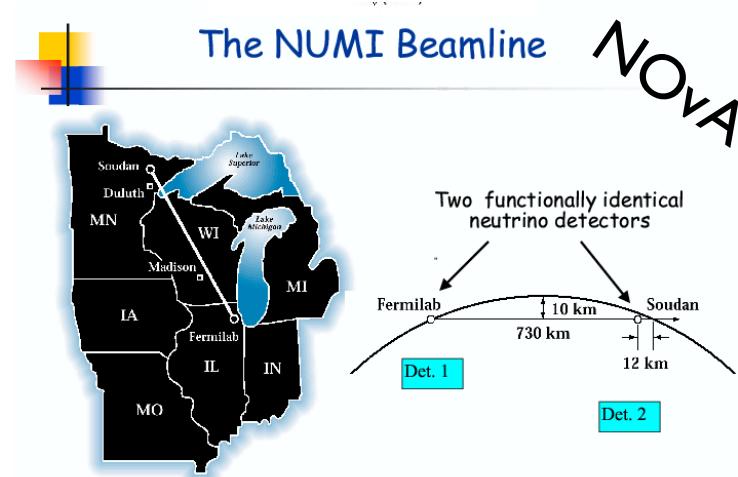
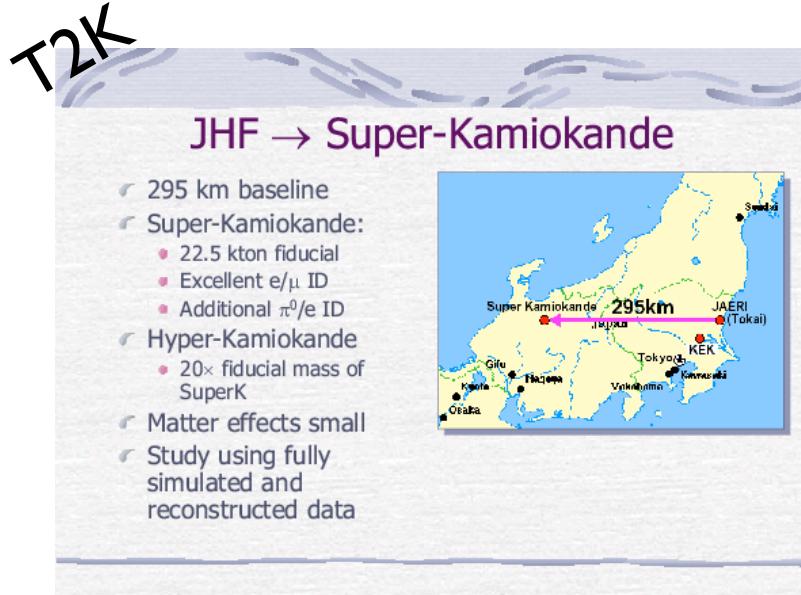
O. Mena + SP
hep-ph/0408070

NOvA:

95% CL Resolution of the Mass Hierarchy



Neutrino v Neutrino Two Expts Different L's and EQUAL $\langle E \rangle / L$'s



2.5 degrees and 14 km
or
2.0 degrees and 12 km

EQUAL <E>/L

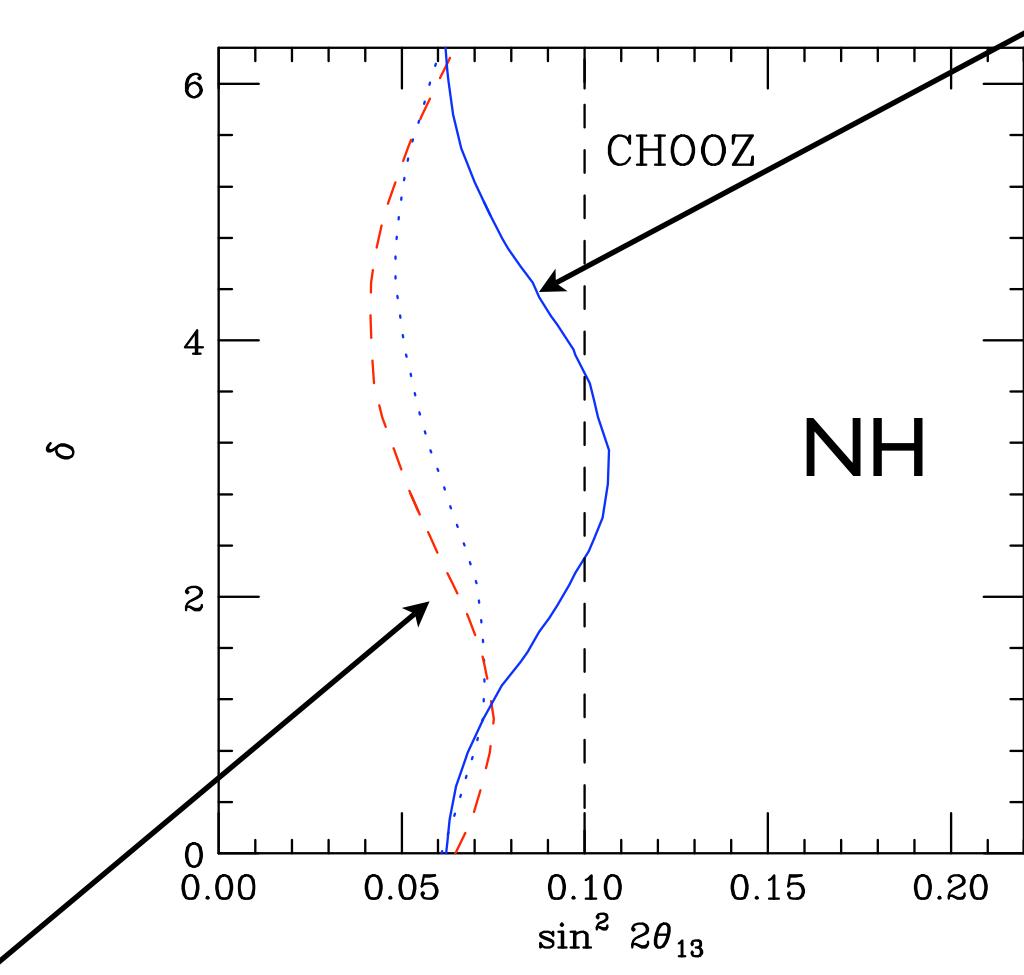
A large, solid orange arrow pointing to the right, indicating the direction of the next section.

but NOT
2.5 degrees and 12 km

Hierarchy Determination

using 2 dof: 90% CL

2.5 deg & 12 km



2.5 deg & 14 km (b) $\Delta m_{31}^2 = +3.0 \times 10^{-3}$ eV²

2.0 deg & 12 km

WHY 2 dof ???
90% at 2 dof
approx
97% at 1 dof

Neutrino v Anti-Neutrino Two Expts. Different L's

T2K

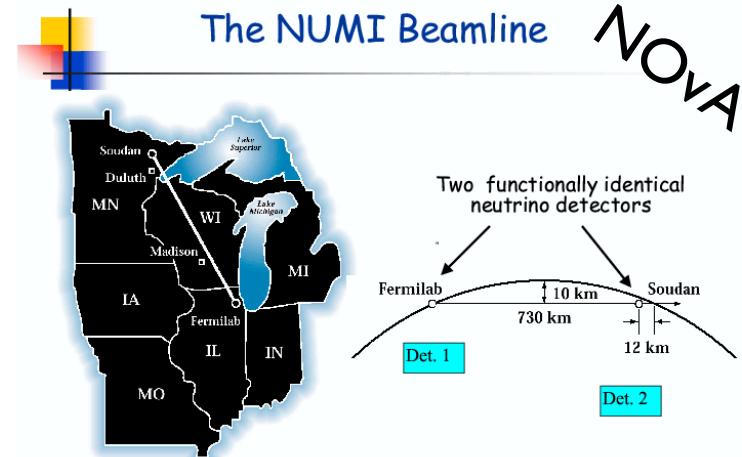
JHF → Super-Kamiokande

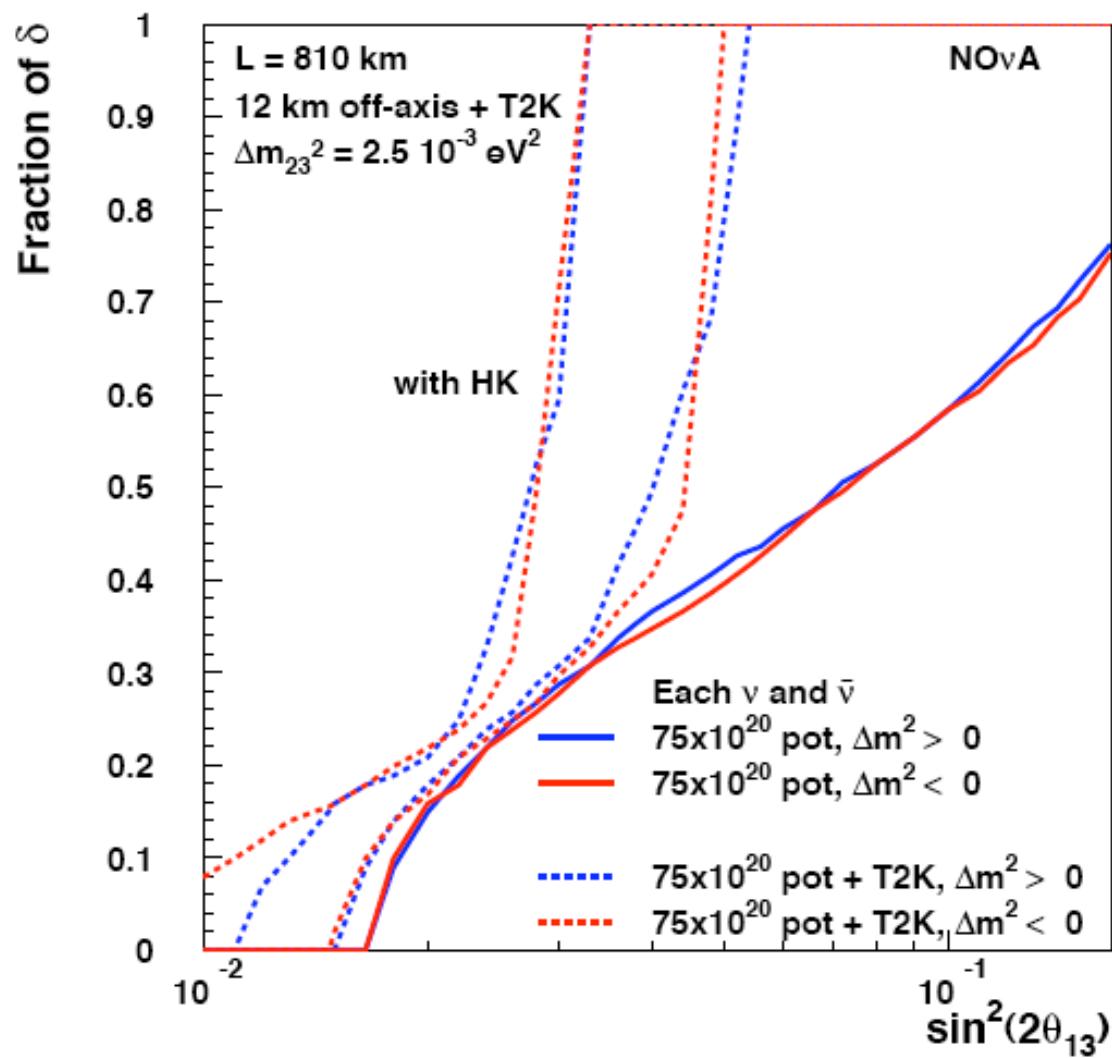
- 295 km baseline
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- Matter effects small
- Study using fully simulated and reconstructed data



The NUMI Beamline

NOVA





NOvA/PD with T2K Phase 2

Spectrum Measurements:

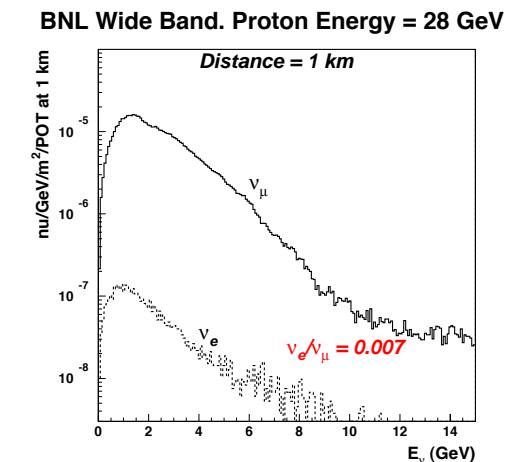
- On Axis
- Off Axis - 2nd Peak

On Axis Beams:



- 28 GeV protons. 1 MW beam power. Horn focussed
- 500 kT water Cherenkov detector.
- baseline > 2500 km. WIPP, Henderson, Homestake

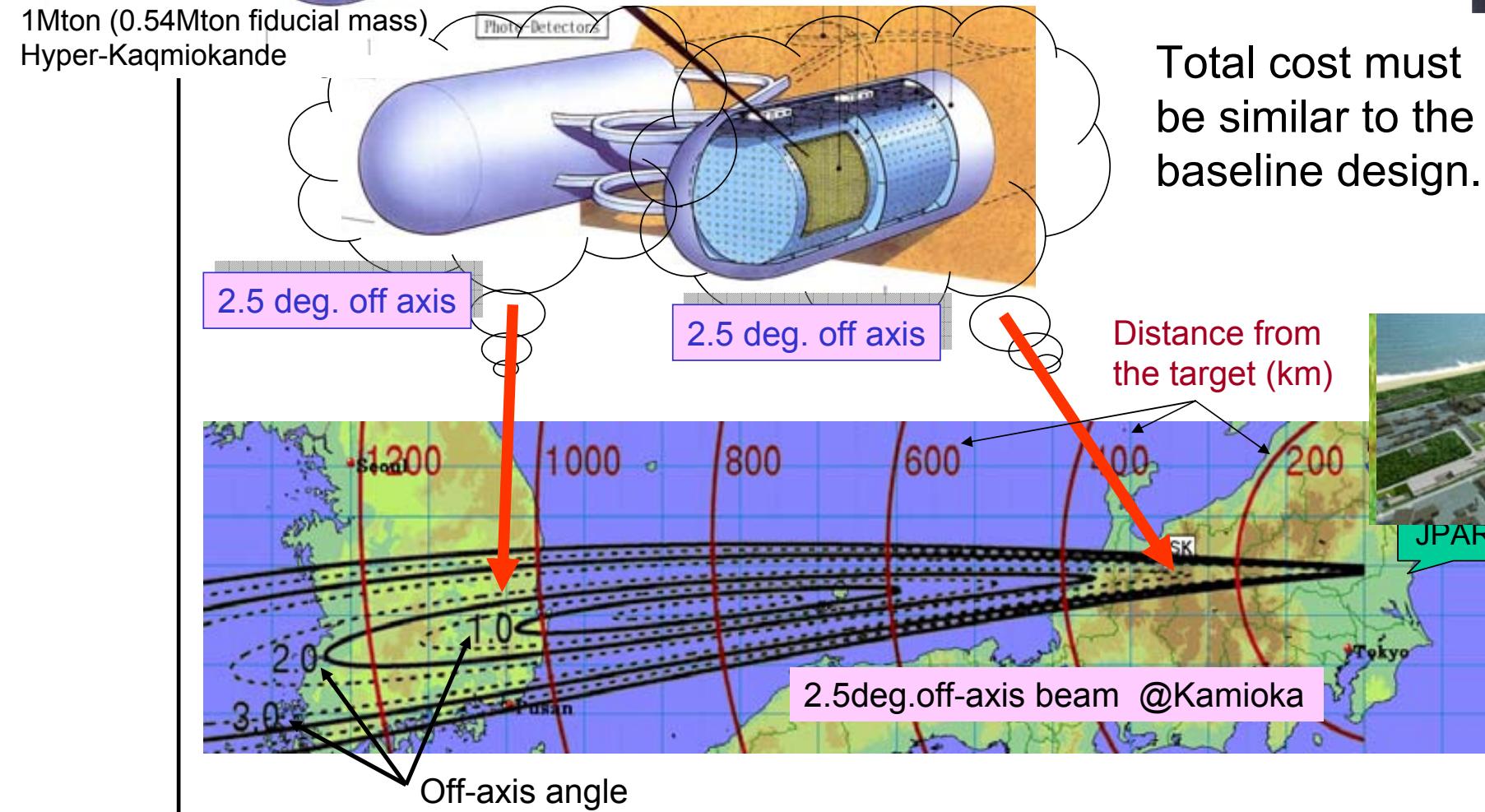
Brookhaven Proposal



Off Axis:



Some recent progress: detector in Korea

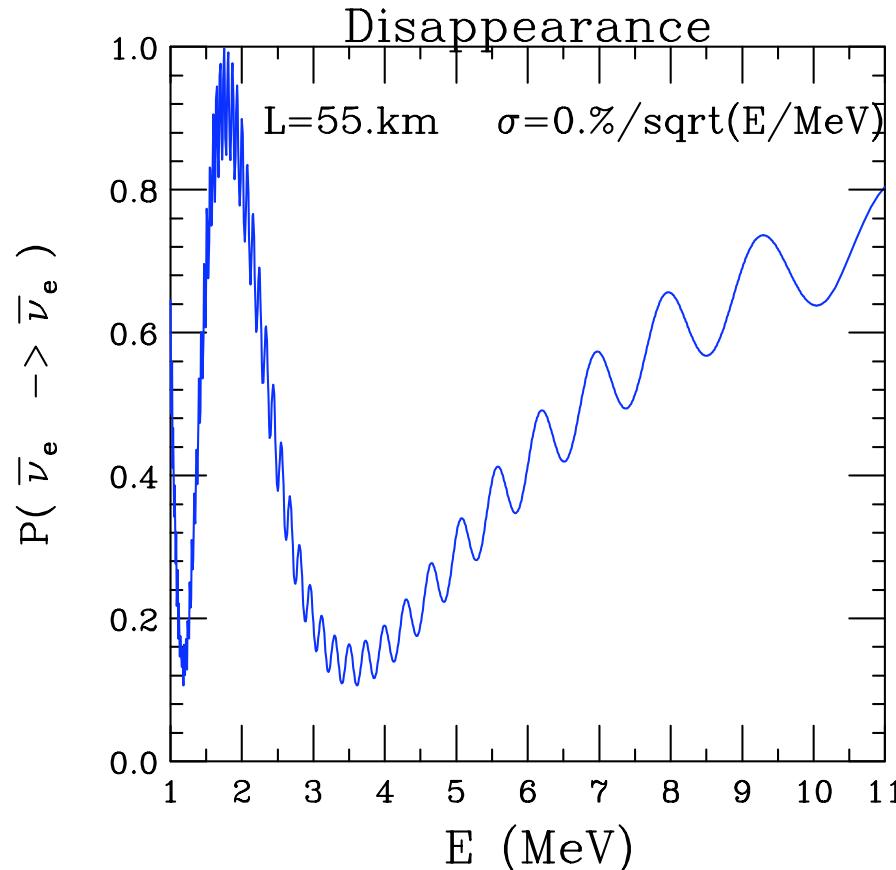


see Kajita talk:

$\bar{\nu}_e$ Disappearance

$$P(\nu_e \rightarrow \nu_e) = 1 - P_{\odot} - \sin^2 2\theta_{13} [c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{32}]$$

$$P_{\odot} \equiv c_{13}^4 \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

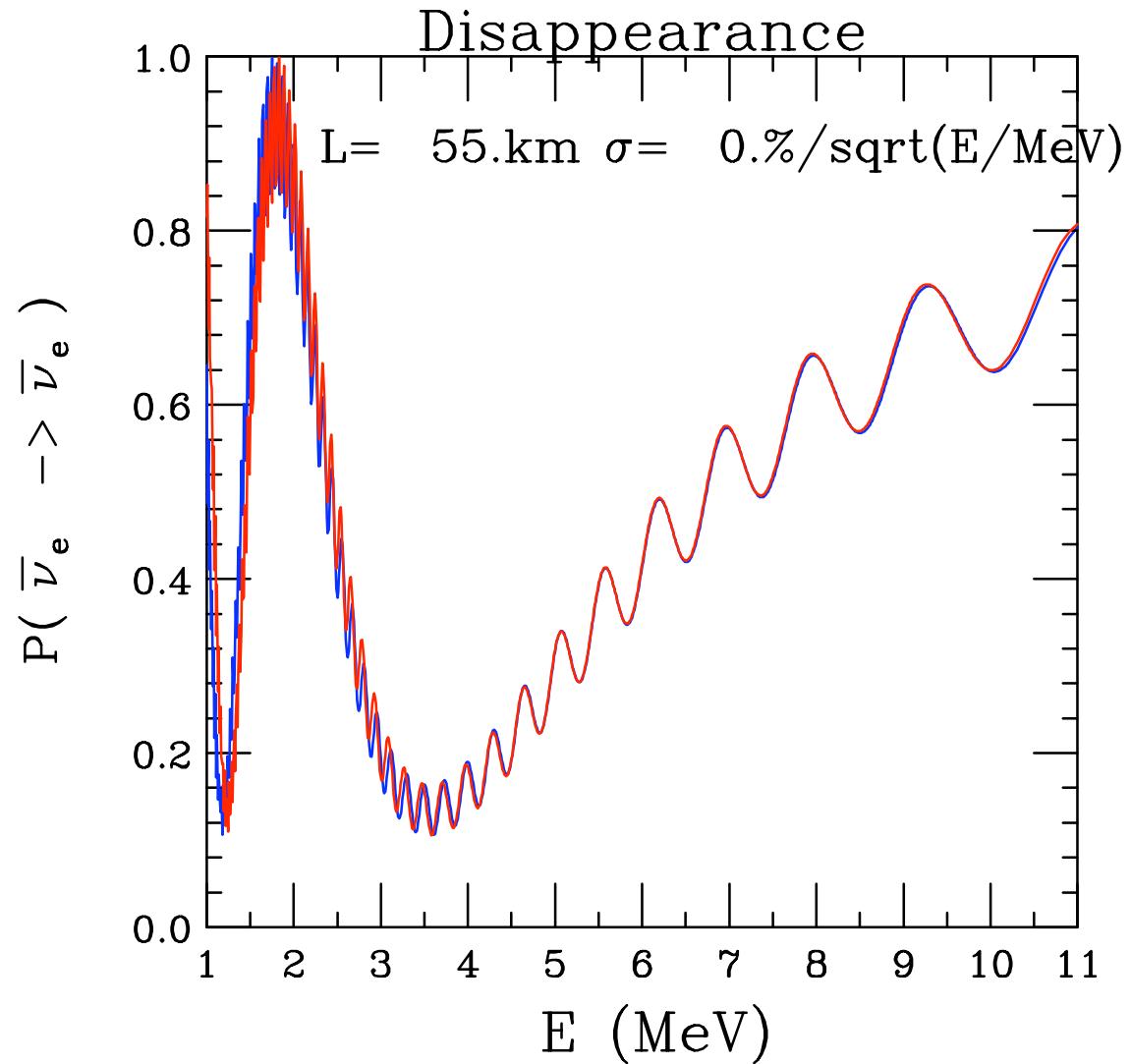


$$\Delta_{21} = \Delta_{31} - \Delta_{32}.$$

$$\Delta_{ij} \equiv \Delta m_{ij}^2 L / 4\hbar c E$$

$$(\Delta m_{ij}^2 \equiv m_i^2 - m_j^2)$$

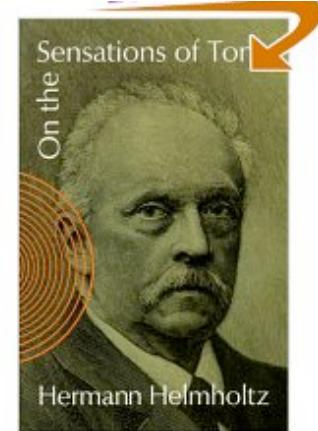
NH v IH :



NH: $|\Delta m_{31}^2| > |\Delta m_{32}^2|$ $(|\Delta_{31}| > |\Delta_{32}|)$

IH: $|\Delta m_{31}^2| < |\Delta m_{32}^2|$ $(|\Delta_{31}| < |\Delta_{32}|)$

$$\begin{aligned}
P(\nu_e \rightarrow \nu_e) &= 1 - P_{\odot} - \sin^2 2\theta_{13} [c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{32}] \\
&= 1 - P_{\odot} \\
&\quad - \frac{1}{2} \sin^2 2\theta_{13} \left\{ 1 + \sqrt{(1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}) \cos(2\bar{\Delta} + \bar{\phi})} \right\}
\end{aligned}$$



where $\bar{\Delta} = \frac{1}{2}(\Delta_{31} + \Delta_{32})$ and $\bar{\phi} = \arctan(\cos 2\theta_{12} \tan \Delta_{21})$

for small Δ_{21} $\Rightarrow \bar{\phi} = \Delta_{21} \cos 2\theta_{12} + \mathcal{O}(\Delta_{21}^3)$

1875

Define $\phi \equiv \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$

then $\frac{d\phi}{dL}|_{L=0} = 0$

thus ϕ doesn't effect the atmospheric oscillation frequency at least for small L.

Rewrite $\cos(2\bar{\Delta} + \bar{\phi}) = \cos(2\Delta_{ee} + \phi)$ then

$$\Delta_{ee} \equiv \frac{1}{2}(\Delta_{31} + \Delta_{32} + \Delta_{21} \cos 2\theta_{21}) = c_{12}^2 \Delta_{31} + s_{12}^2 \Delta_{32}$$

since $\Delta_{21} = \Delta_{31} - \Delta_{32}$.

$$P(\nu_e \rightarrow \nu_e) = 1 - P_\odot$$

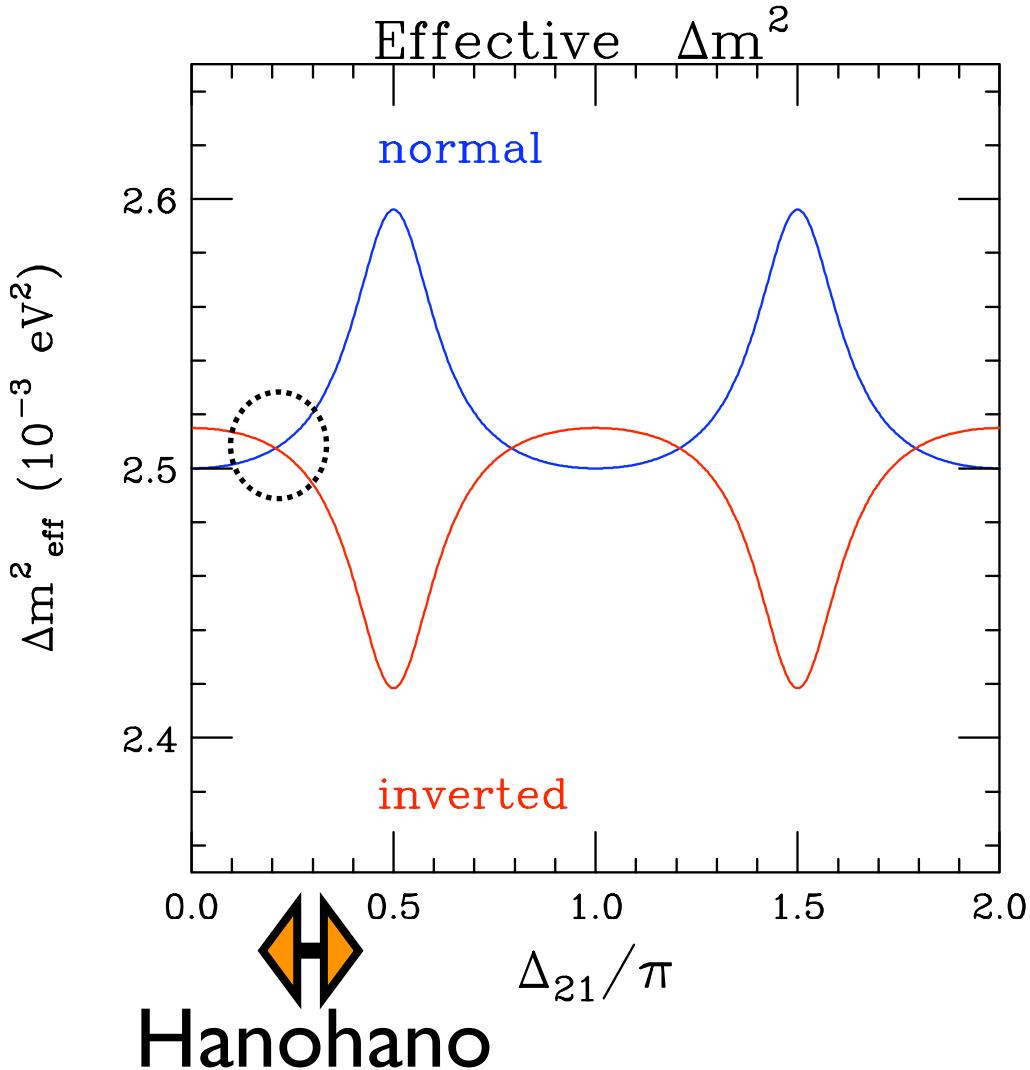
$$-\frac{1}{2} \sin^2 2\theta_{13} \left\{ 1 + \sqrt{(1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21}) \cos(2\Delta_{ee} \pm \phi)} \right\}$$

NH (+) and IH (-): $\Delta_{ee} \equiv \Delta m_{ee}^2 L / 4E$, $\Delta m_{ee}^2 = c_{12}^2 |\Delta m_{31}^2| + s_{12}^2 |\Delta m_{32}^2|$

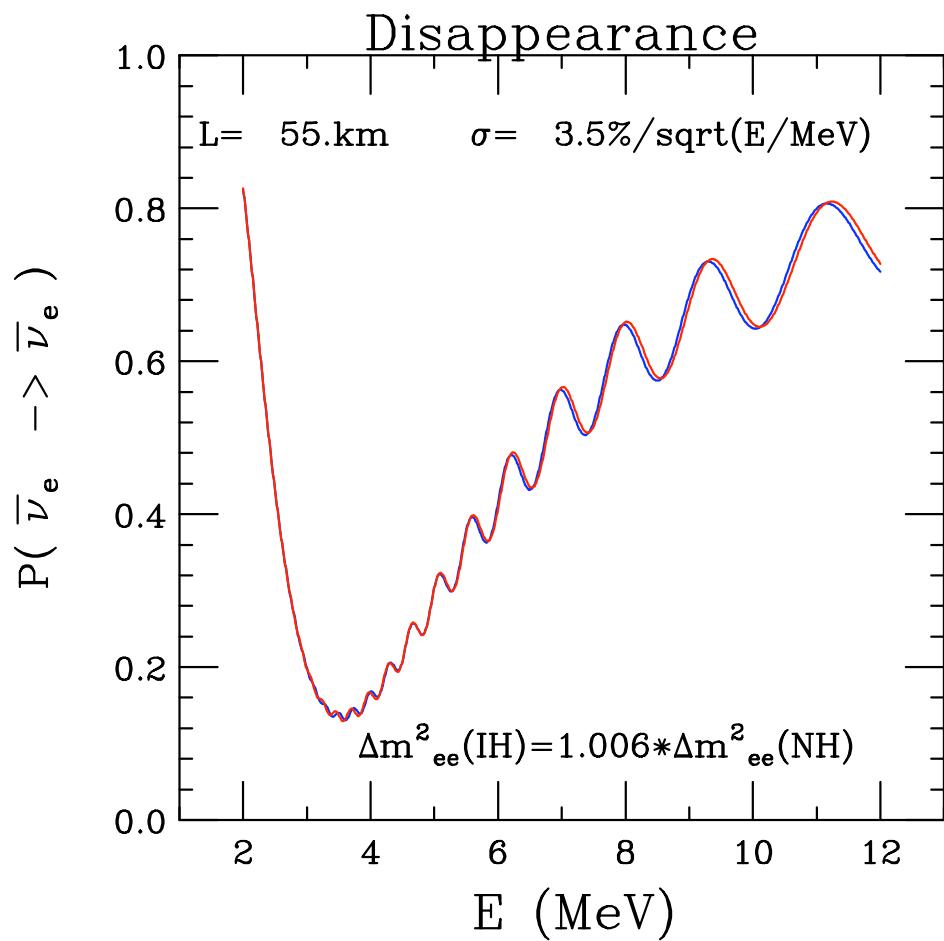
$$\phi \equiv \arctan(\cos 2\theta_{12} \tan \Delta_{21}) - \Delta_{21} \cos 2\theta_{12}$$

$$\Delta m_{eff}^2 \equiv \frac{d (2\Delta_{ee} \pm \phi_\odot)}{d L / 2E}$$

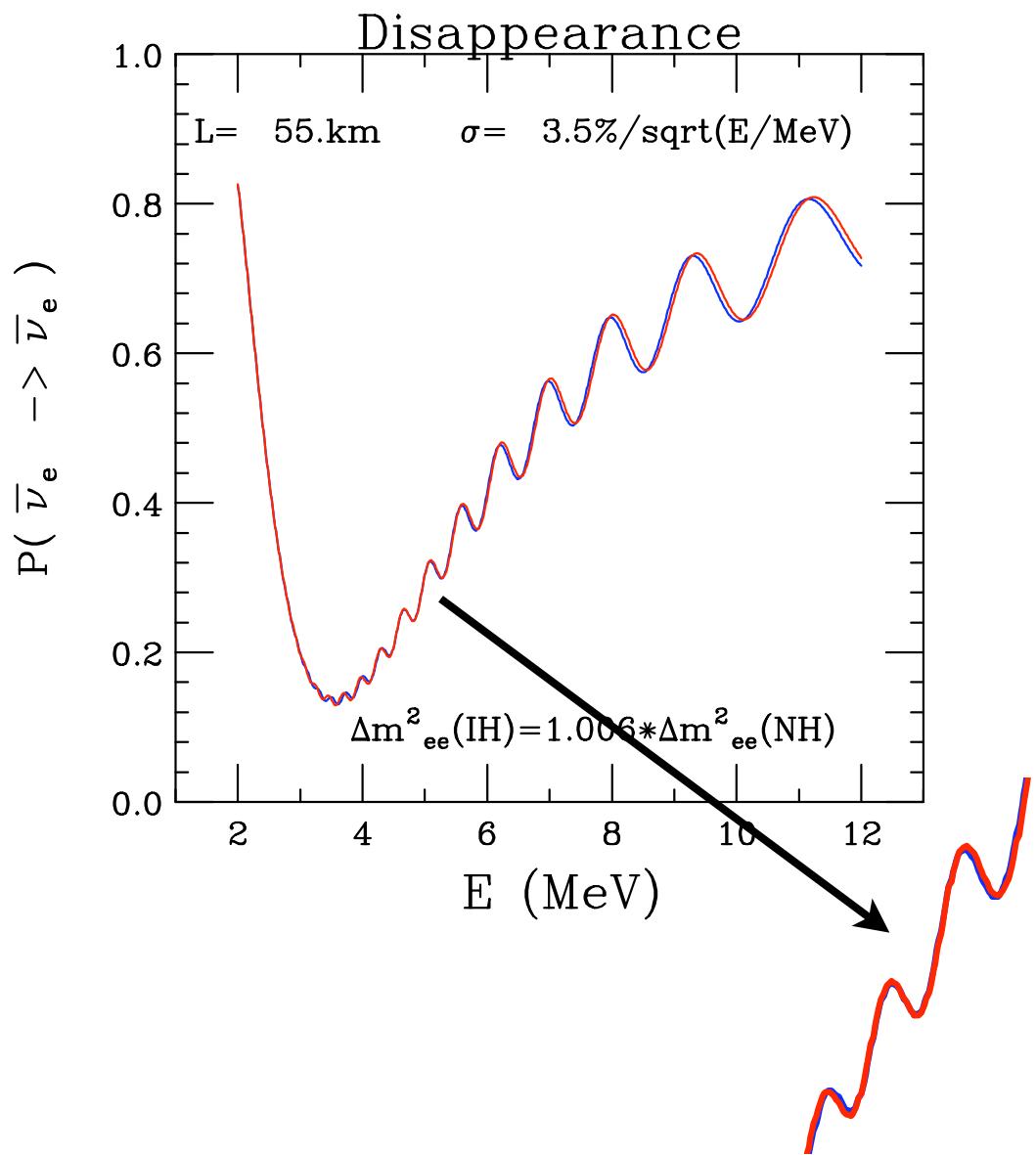
$$\begin{aligned}
\Delta m_{eff}^2 &\equiv \frac{d (2\Delta_{ee} \pm \phi_\odot)}{d L/2E} \\
&= \Delta m_{ee}^2 \pm \frac{1}{2} \Delta m_{21}^2 \cos 2\theta_{12} \frac{\sin^2 2\theta_{12} \sin^2 \Delta_{21}}{(1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21})}
\end{aligned}$$



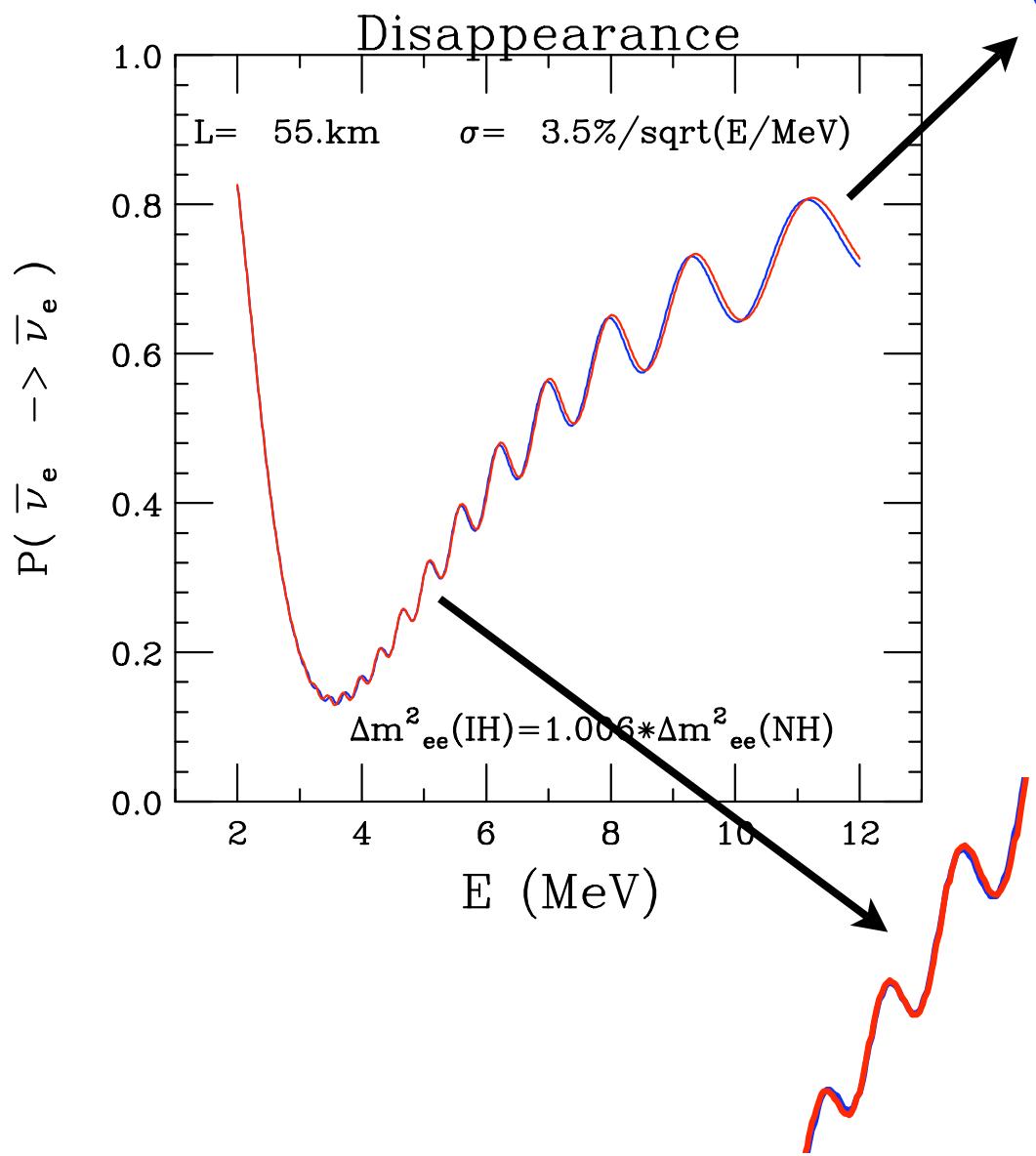
NH v IH :



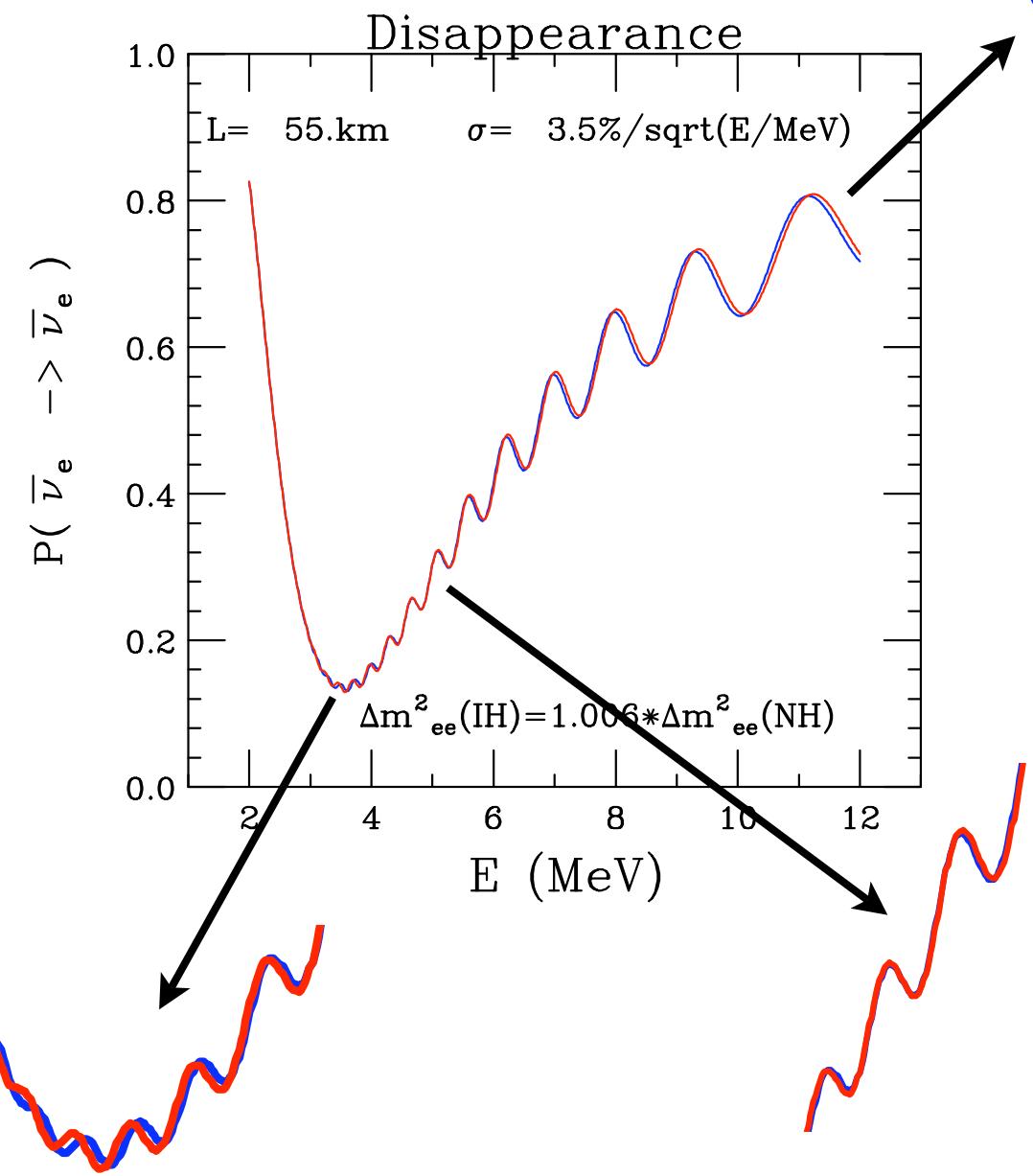
NH v IH :



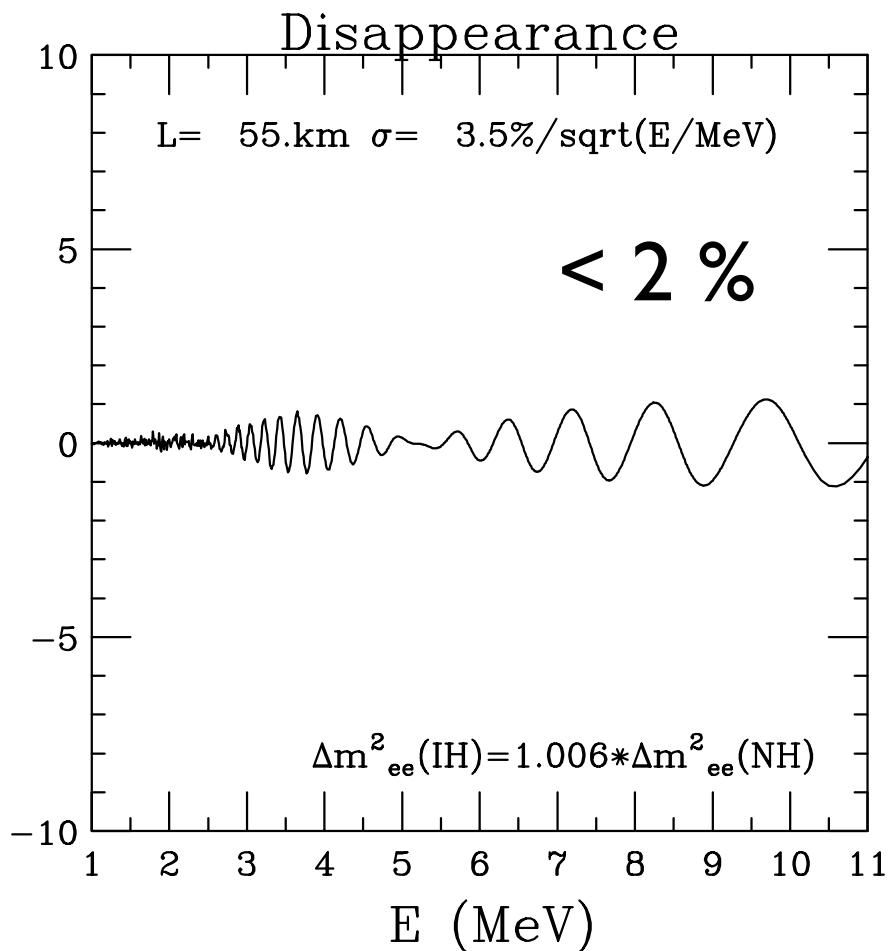
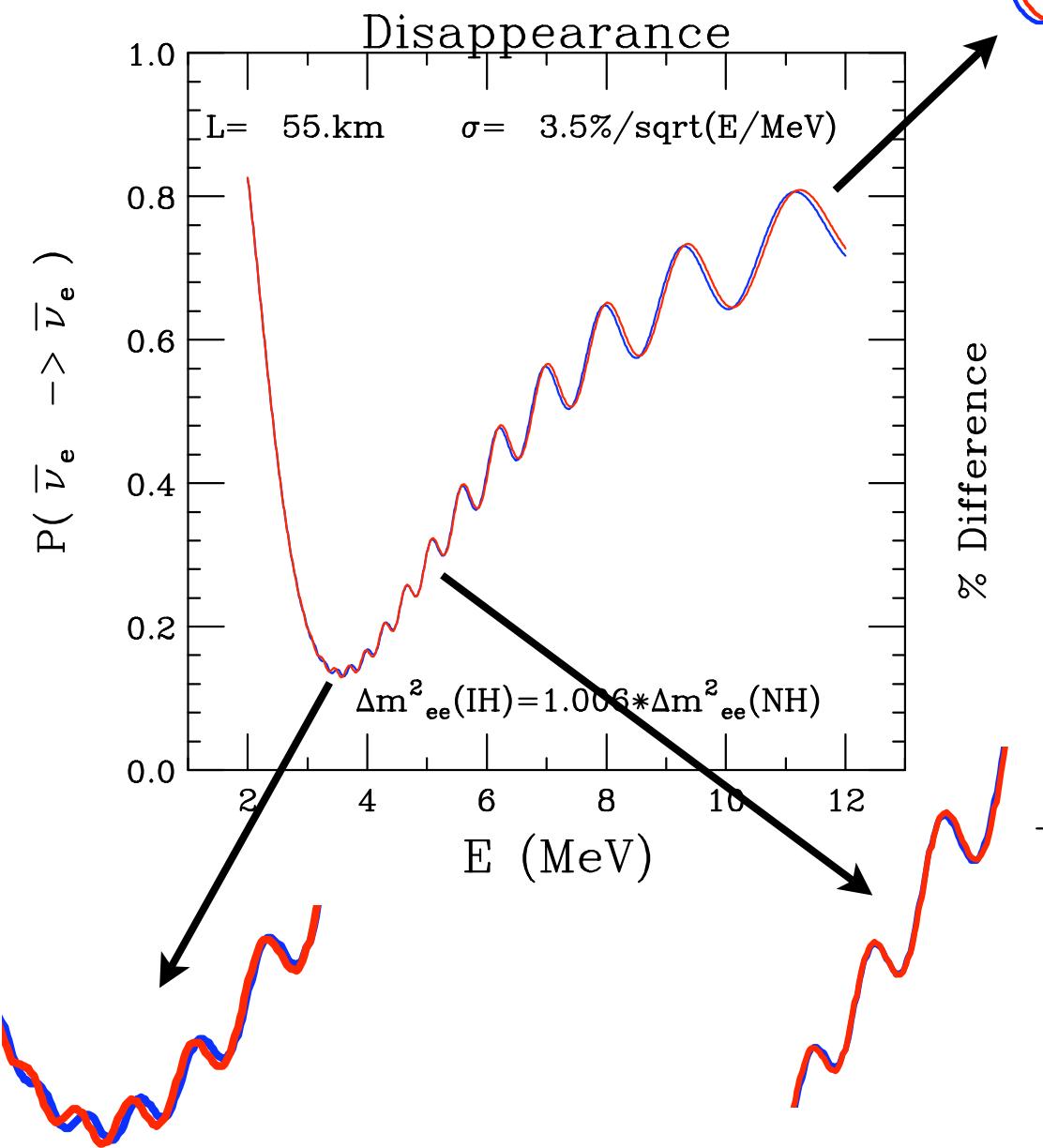
NH v IH:



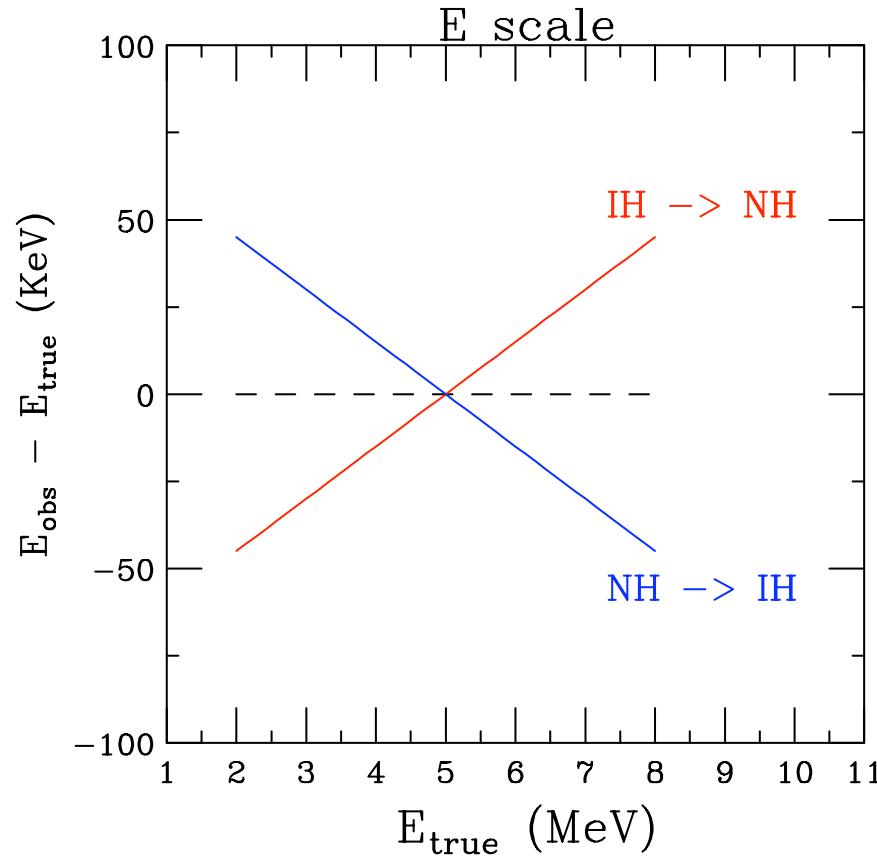
NH v IH:



NH v IH:



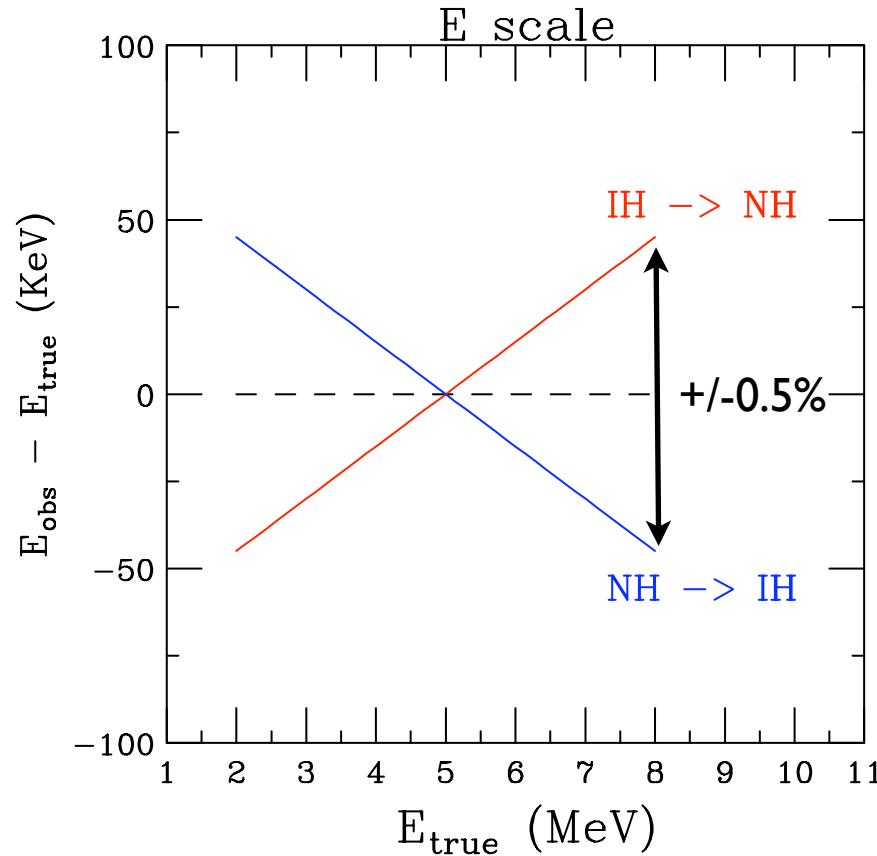
Uncertainty in E scale ??? between 2 and 8 MeV !!!



$$E_{obs} = E_{true} + 0.015 \times (E_{true} - 5)$$

$$E_{obs} = E_{true} - 0.015 \times (E_{true} - 5)$$

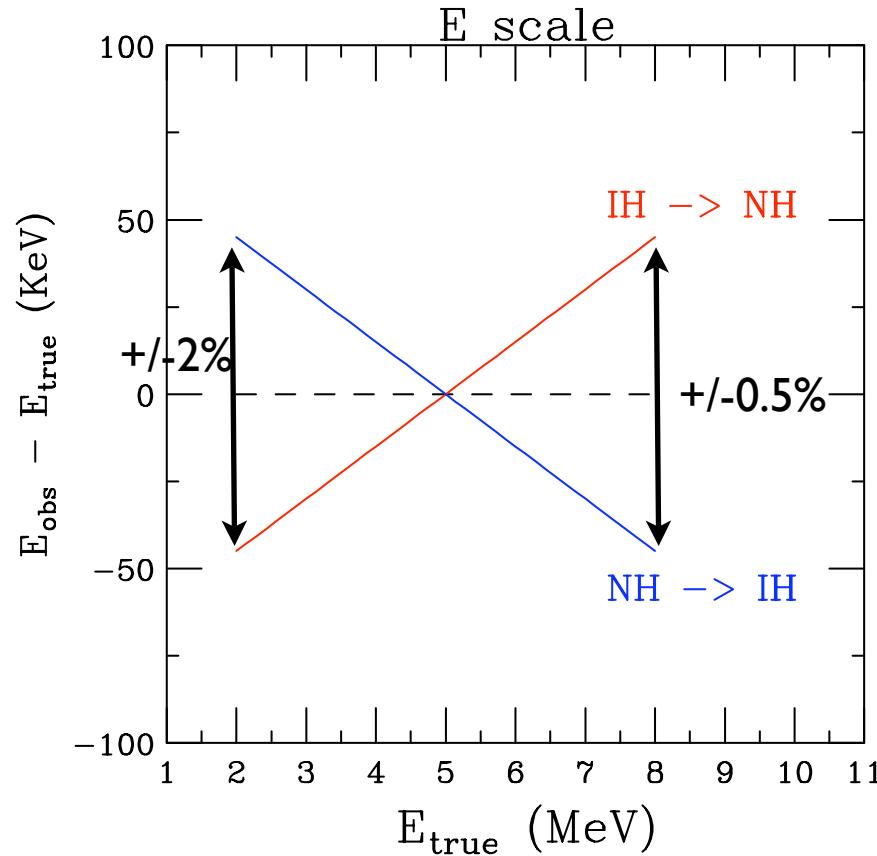
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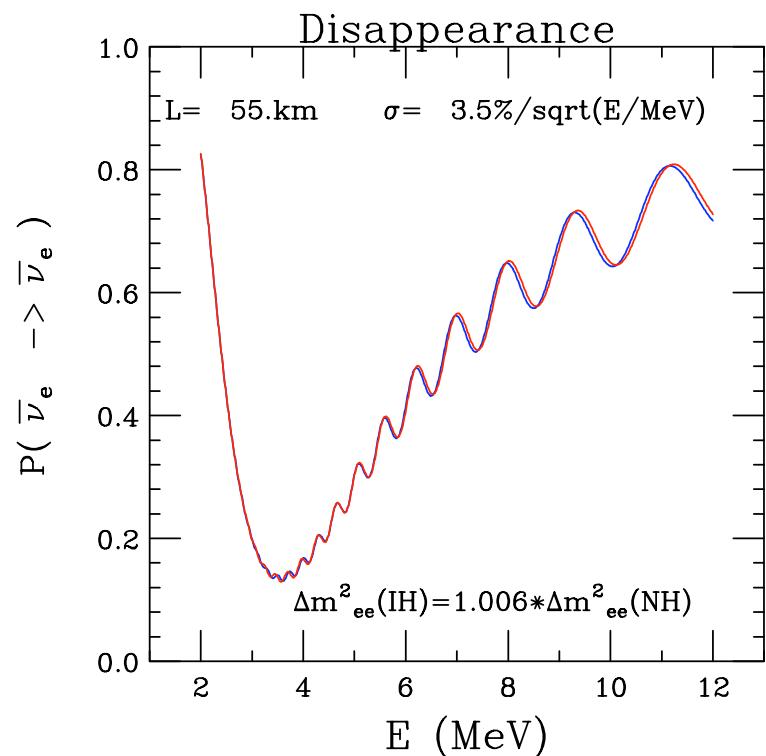
Uncertainty in E scale ??? between 2 and 8 MeV !!!



$$E_{obs} = E_{true} + 0.015 \times (E_{true} - 5)$$

$$E_{obs} = E_{true} - 0.015 \times (E_{true} - 5)$$

Distinguish NH from IH:



- Over 2 to 8 MeV Window
- High Quality data
- +
- Large Statistics

Magic of Fourier Transforms: JL talk

Conclusions:

- Determination of Hierarchy is Challenging
- NOvA and T2K statistics limited
- Help Needed!!!
- Large, High Quality data sets from Reactor - Hanohano
- Hanohano Limit independent of value of the CP violating phase.

